The Performance of Fuzzy Operators on Fuzzy Classification of Urban Land Covers

Z. Islam and G. Metternicht

Abstract
The research discussed in this paper evaluates the performance of selected fuzzy operators (e.g., maximum, minimum, algebraic sum, algebraic product, and gamma operators) for integrating fuzzy membership values associated with multiple spectral bands for mapping the complex spatial mixture that characterises urban land covers. Accordingly, a supervised classification approach based on the fuzzy c-means algorithm was implemented to generate fuzzy memberships of selected bands (1, 3, 4 and 7) of a Landsat-7 ETM+ image that provided the highest spectral separability among different urban land covers (e.g., forest, grassland, urban, and dense urban) as determined by a transformed divergence analysis. Maps resulting from the application of each fuzzy operator were evaluated against field data. The results show that the fuzzy algebraic product and the fuzzy gamma operators (0.1 to 0.8) are optimal for integrating the fuzzy memberships of selected urban land covers on multi-band data sets, as they exhibited a $K_{hub}$ statistic of 75 percent as compared to a $K_{stat}$ statistic of 59 percent, 64 percent and 71 percent for maximum, minimum and fuzzy algebraic sum, respectively.

Introduction
Thematic mapping of urban landscapes using remote sensing images is challenging due to the heterogeneity of the land covers. Typically, urban land covers consist of a complex mixture and spatial arrangement of man-made and natural land cover types; determining that reflectance values associated with individual pixels are often the result of the interaction of more than one surface material present within the area covered by a pixel (Forster, 1985). In turn, this results in the occurrence of mixed pixels, especially when working with remotely sensed images of coarse spatial and/or spectral resolution. Conventional remote sensing analysis that relies on the assumption that a training area is composed of unique, internally homogenous classes works poorly for urban land cover mapping. This is a limitation of the traditional parametric methods for deriving urban land covers (Zhang and Foody, 1996), and therefore, the performance of non-parametric classifiers or fuzzy classifiers based on fuzzy set theory for mapping heterogeneous urban land covers needs to be investigated.

In a fuzzy classification, a pixel is assigned a value representing its grade to each possible individual (e.g., land cover class) in the universe of discourse (e.g., urban areas). This grade corresponds to the degree to which that pixel belongs to its associated land cover class and it is termed fuzzy membership value (FMV) (Lowell, 1994). To some extent, the FMVs reflect the land cover composition of a mixed pixel which enables a more accurate and realistic representation of land covers, overcoming some of the assumptions of the conventional classifications (Fisher and Pathirana, 1990; Foody and Cox, 1994). The FMVs can be derived from a range of classifiers for example, the fuzzy c-means algorithm (Bezdek et al., 1984), supervised non-parametric classifier (Skidmore and Turner, 1988) and maximum likelihood classifier (Foody et al., 1992). The advantage of fuzzy c-means algorithm is that it can be used for either supervised (e.g., Cannon et al., 1986a) or unsupervised classification (e.g., Key et al., 1989).

For multi-spectral satellite images, FMVs associated to predetermined informational classes can be generated for each band. According to An et al. (1991), the integration of FMVs of more than one layer of information sometimes compensates each other in the final result. However, the question often arises as to how to select the most suitable combination of bands, which are to be fuzzified and integrated. Assuming a Landsat-7 ETM+ data set is used without undertaking a band selection, we would end up creating a total of 28 information layers (7 bands + 4 land cover types). This result in a cumbersome number of layers to be integrated for obtaining fuzzy urban land cover maps which, in addition, may contain redundant information. To this end, feature selection techniques such as the transformed divergence analysis can be employed for selecting the most appropriate sensor’s band, or set of bands, to be fuzzified as part of the procedure for deriving urban land cover maps using fuzzy supervised classification (Metternicht, 1996; Metternicht and Zinck, 1997 and 1998).

Fuzzy operators based on fuzzy set theory (Zadeh, 1965) can be used to integrate the fuzzy memberships of selected bands, as these operators allow manipulating and processing incomplete and/or imprecise information to obtain the most reasonable output. Mohan et al. (2000) applied fuzzy operators to integrate the fuzzy memberships of different land uses as derived from multitemporal images, reporting a significant increase in classification accuracy. Though research has been carried out applying a variety of fuzzy operators for integrating fuzzy membership of geophysical and geological data sets (An et al., 1991; Moon et al., 1991), not many applications have focussed on the use of fuzzy operators for integrating fuzzy memberships of urban land covers, as computed on various bands of multispectral satellite image. Thus, the objective of this study is to identify an optimal fuzzy operator for integrating fuzzy membership values associated with multiple spectral bands for mapping the major land covers that characterise urban landscapes.

A fuzzy classification technique is applied to generate FMVs of four major urban land cover classes in selected Landsat-7 ETM+ bands. Fuzzy operators (e.g., maximum, minimum, algebraic sum, algebraic product, and gamma operators with $\gamma$ values ranging from 0.1 to 0.95) are applied to how to select the most suitable combination of bands, which are to be fuzzified and integrated. Assuming a Landsat-7 ETM+ data set is used without undertaking a band selection, we would end up creating a total of 28 information layers (7 bands + 4 land cover types). This result in a cumbersome number of layers to be integrated for obtaining fuzzy urban land cover maps which, in addition, may contain redundant information. To this end, feature selection techniques such as the transformed divergence analysis can be employed for selecting the most appropriate sensor’s band, or set of bands, to be fuzzified as part of the procedure for deriving urban land cover maps using fuzzy supervised classification (Metternicht, 1996; Metternicht and Zinck, 1997 and 1998).

For multi-spectral satellite images, FMVs associated to predetermined informational classes can be generated for each band. According to An et al. (1991), the integration of FMVs of more than one layer of information sometimes compensates each other in the final result. However, the question often arises as to how to select the most suitable combination of bands, which are to be fuzzified and integrated. Assuming a Landsat-7 ETM+ data set is used without undertaking a band selection, we would end up creating a total of 28 information layers (7 bands + 4 land cover types). This result in a cumbersome number of layers to be integrated for obtaining fuzzy urban land cover maps which, in addition, may contain redundant information. To this end, feature selection techniques such as the transformed divergence analysis can be employed for selecting the most appropriate sensor’s band, or set of bands, to be fuzzified as part of the procedure for deriving urban land cover maps using fuzzy supervised classification (Metternicht, 1996; Metternicht and Zinck, 1997 and 1998).

Fuzzy operators based on fuzzy set theory (Zadeh, 1965) can be used to integrate the fuzzy memberships of selected bands, as these operators allow manipulating and processing incomplete and/or imprecise information to obtain the most reasonable output. Mohan et al. (2000) applied fuzzy operators to integrate the fuzzy memberships of different land uses as derived from multitemporal images, reporting a significant increase in classification accuracy. Though research has been carried out applying a variety of fuzzy operators for integrating fuzzy membership of geophysical and geological data sets (An et al., 1991; Moon et al., 1991), not many applications have focussed on the use of fuzzy operators for integrating fuzzy memberships of urban land covers, as computed on various bands of multispectral satellite image. Thus, the objective of this study is to identify an optimal fuzzy operator for integrating fuzzy membership values associated with multiple spectral bands for mapping the major land covers that characterise urban landscapes.

A fuzzy classification technique is applied to generate FMVs of four major urban land cover classes in selected Landsat-7 ETM+ bands. Fuzzy operators (e.g., maximum, minimum, algebraic sum, algebraic product, and gamma operators with $\gamma$ values ranging from 0.1 to 0.95) are applied to how to select the most suitable combination of bands, which are to be fuzzified and integrated. Assuming a Landsat-7 ETM+ data set is used without undertaking a band selection, we would end up creating a total of 28 information layers (7 bands + 4 land cover types). This result in a cumbersome number of layers to be integrated for obtaining fuzzy urban land cover maps which, in addition, may contain redundant information. To this end, feature selection techniques such as the transformed divergence analysis can be employed for selecting the most appropriate sensor’s band, or set of bands, to be fuzzified as part of the procedure for deriving urban land cover maps using fuzzy supervised classification (Metternicht, 1996; Metternicht and Zinck, 1997 and 1998).

Fuzzy operators based on fuzzy set theory (Zadeh, 1965) can be used to integrate the fuzzy memberships of selected bands, as these operators allow manipulating and processing incomplete and/or imprecise information to obtain the most reasonable output. Mohan et al. (2000) applied fuzzy operators to integrate the fuzzy memberships of different land uses as derived from multitemporal images, reporting a significant increase in classification accuracy. Though research has been carried out applying a variety of fuzzy operators for integrating fuzzy membership of geophysical and geological data sets (An et al., 1991; Moon et al., 1991), not many applications have focussed on the use of fuzzy operators for integrating fuzzy memberships of urban land covers, as computed on various bands of multispectral satellite image. Thus, the objective of this study is to identify an optimal fuzzy operator for integrating fuzzy membership values associated with multiple spectral bands for mapping the major land covers that characterise urban landscapes.

A fuzzy classification technique is applied to generate FMVs of four major urban land cover classes in selected Landsat-7 ETM+ bands. Fuzzy operators (e.g., maximum, minimum, algebraic sum, algebraic product, and gamma operators with $\gamma$ values ranging from 0.1 to 0.95) are applied to how to select the most suitable combination of bands, which are to be fuzzified and integrated. Assuming a Landsat-7 ETM+ data set is used without undertaking a band selection, we would end up creating a total of 28 information layers (7 bands + 4 land cover types). This result in a cumbersome number of layers to be integrated for obtaining fuzzy urban land cover maps which, in addition, may contain redundant information. To this end, feature selection techniques such as the transformed divergence analysis can be employed for selecting the most appropriate sensor’s band, or set of bands, to be fuzzified as part of the procedure for deriving urban land cover maps using fuzzy supervised classification (Metternicht, 1996; Metternicht and Zinck, 1997 and 1998).
to integrate the FMVs determined for each urban land cover on the multi-spectral data set. The performance of the land cover maps generated by various fuzzy operators is assessed using $K_{hat}$ statistic. A brief review of the fuzzy c-means algorithm used for computing fuzzy memberships of land covers, the integration of these memberships using fuzzy operators, and defuzzification of the classified data is presented in the first section of the article. The methodology, including transformed divergence analysis, is outlined in a following section, followed by the discussion of results and conclusions.

**Derivation of Fuzzy Urban Land Cover Maps**

The conceptual model devised for the generation of fuzzy urban land cover maps is composed of four main steps, as shown in Figure 1: (1) Selection of the best set of Landsat-7
ETM+ bands using transformed divergence analysis; (2) Computation of fuzzy membership values for each land cover type on the multi-band Landsat-7 ETM+ data set; (3) Integration of fuzzy membership values for land cover types A, B, C, . . . N in Landsat-7 ETM+ band 1, 2, . . . , n using different fuzzy operators; (4) Defuzzification of the output map and accuracy evaluation. The theoretical basis of the model is presented hereafter.

**Fuzzy c-Means Algorithm**

As noted in the first section, there are several methods for computing FMVs (Bezdek et al., 1984; Skidmore and Turner, 1988; Foody et al., 1992), but a common approach is the fuzzy c-means algorithm. The fuzzy c-means clustering algorithm is based on the concept of fuzzy sets as proposed by Zadeh (1965).

The membership function in the fuzzy c-means clustering is generated for each pixel showing similarity of pixel membership to each predetermined class. Thus, a pixel will have a fuzzy membership to every informational class. Membership values close to one signify a high degree of similarity between a pixel and fuzzy class, while membership values close to zero imply little similarity between the pixel and that class. According to Bezdek et al. (1984), an optimal fuzzy-c partition is created through minimisation of the generalised least-squared errors which can be represented as shown in Equation 1:

$$J_m(U,v) = \sum_{i=1}^{N} \left( \sum_{j=1}^{C} \mu_{ij}^{m} d_{ij}^{2} \right)$$

where $J_m$ is the objective function, $U$ represents the fuzzy c-partition, $v$ represents the cluster centers, $\mu_{ik}$ is the membership in cluster $i$, $d_{ik}$ is the distance between each observation and the class mean, $c$ is the number of clusters, $N$ is number of observations, and $m$ is weighting exponent; $1 \leq m < \infty$.

The weighting exponent $m$ controls the relative weights placed on each of the squared errors $d_{ik}^2$, and it is used to determine the membership function. As $m \rightarrow 1$, partitions that minimize $J_m$ are increasingly hard and become completely hard at $m = 1$. This is usually known as the k-means clustering (McBratney and Moore, 1985). Consequently, an increasing $m$ tends to degrade membership towards the fuzziest state, and each entry of optimal $U_i$ for $J_m$ approaches $(1/c)$ as $m \rightarrow \infty$. Thus, the weighting exponent $m$ controls the extent of membership sharing between fuzzy clusters in $X$.

McBratney and Moore (1985) tested a range of values for $m$, and found an $m$ value of approximately 2 to be optimal. There is a lack of theoretical basis for choosing $m$, but typically $1.1 \leq m \leq 5$ is reported as the most useful range of values (Cannon et al., 1986b). Key et al. (1989) reported that the range of useful values of $m$ is 1 to 30, while for most data, $1.5 \leq m \leq 3.0$ exhibited good results. For a particular weighting component, Bezdek (1981) showed mathematically that in the least squared error, $J_m$ minimises only for optimal values of fuzzy membership and the mean value ($U_{i}$, $v$), thus achieving the optimal fuzzy c-clustering of $X$. Therefore, optimal values of the fuzzy membership and mean values ($U_{i}$, $v$) are needed to obtain the least squared error.

**Fuzzy Operators for Integration of Fuzzy Membership**

Among the fuzzy operators, the following are the most frequently applied to integrate the memberships of various data sets or information layers (An et al., 1991; Bonham-Carter, 1994).

**Fuzzy AND**

Fuzzy AND is a logical intersection which combines the fuzzy memberships of two or more layers (e.g., Landsat-7 ETM+ bands) using a fuzzy minimum operator. It can be defined as

$$\mu_{\text{land cover } A} = \min (\mu_{A1}, \mu_{A2}, \mu_{A3}, \ldots, \mu_{An})$$

where $\mu_{A1}, \mu_{A2}, \mu_{A3}, \ldots, \mu_{An}$ represent the membership values of land cover $A$ in the Landsat-7 ETM+ bands 1, 2, 3, . . . , n, respectively at a particular location. This indicates that the operator generates a map displaying minimum membership values for each location. Thus, the resultant image is a conservative estimation of a set of memberships that tend to produce small values (Bonham-Carter, 1994).

**Fuzzy OR**

Fuzzy OR combines the fuzzy memberships in land cover $A$ from two or more layers (e.g., Landsat-7 ETM+ bands) using a fuzzy maximum operator. The output membership values are controlled by the maximum membership values of the input bands for any particular location. This can be defined as

$$\mu_{\text{land cover } A} = \max (\mu_{A1}, \mu_{A2}, \mu_{A3}, \ldots, \mu_{An})$$

**Fuzzy Algebraic Product**

The fuzzy algebraic product is a modified intersection which combines remote sensing data by multiplying the membership of land cover $A$ on each selected band, as defined by the following equation:

$$\mu_{\text{land cover } A} = \prod_{i=1}^{n} \mu_{Ai}$$

where $\mu_{Ai}$ is the fuzzy membership in land cover $A$ for the $i$th band and $i = 1, 2, 3, \ldots, n$ bands are to be combined. With this operator, the combined fuzzy membership values tend to be very small due to the effect of multiplying several numbers lower than one. The output is always smaller than, or equal to, the smallest contributing membership value, and it is therefore “decreasive”. Although the fuzzy algebraic product gives an output that is decreasive in nature, it does utilise every membership value to produce the result, unlike the fuzzy minimum (e.g., AND or maximum (OR) operators (Bonham-Carter, 1994)).

**Fuzzy Algebraic Sum**

The fuzzy algebraic sum is a modified union, which is expressed as a probabilistic sum (Zimmermann, 1991). For a fuzzy membership, $\mu_{Ai}$ in land cover $A$, where $i$th bands are to be combined, the probabilistic sum can be simplified by the following equation:

$$\mu_{\text{land cover } A} = 1 - \prod_{i=1}^{n} (1 - \mu_{Ai})$$

The combined fuzzy membership value of land cover $A$ is always larger or equal to the largest contributing fuzzy membership value. Thus, the effect is increasive. If two or more pieces of evidence favour a hypothesis (e.g., the possibility of a pixel to be classified as land cover $A$, as expressed by fuzzy membership values in the Landsat-7 ETM+ bands 1 to $n$) it reinforces one another, and the combined evidence is more supportive than either pieces individually (Bonham-Carter, 1994).

**Fuzzy Gamma Operation**

The gamma ($\gamma$) operator is defined in terms of the fuzzy algebraic product and the fuzzy algebraic sum by the following equation:

$$\mu_{\text{land cover } A} = (\text{Fuzzy algebraic sum})^\gamma \cdot (\text{Fuzzy algebraic product})^{1-\gamma}$$

where $\gamma$ is a parameter chosen in the range $[0, 1]$ (Zimmermann, 1985). Equation 6 indicates that when $\gamma = 1$
equals 1, the combined fuzzy membership of land cover \( A \) is the same as the fuzzy algebraic sum whereas the combination equals the fuzzy algebraic product when \( \gamma \) equals 0. Clearly, the choice of \( \gamma \) compromises between the “increasive” tendencies of the fuzzy algebraic sum and the “decreaseive” effect of the fuzzy algebraic product.

**Defuzzification of the Fuzzy Classified Data**

Defuzzification provides a basis to carry out the accuracy assessment of the fuzzy classified data, which is problematic due to the non-crisp nature of data characterising the fuzziness in the context of objective ground truth (Zhang and Goodchild, 2002). It is achieved by converting fuzzy maps as unions of defuzzified areal classes and fuzzy boundaries, using subsets exceeding or falling below a suitably chosen quantitative \( \alpha \)-cut level (Zadeh, 1968). In a typical defuzzification of the fuzzy classified data, the pixels are assigned to the classes at which they have the maximum possibility to belong, as measured by specific class membership functions (Zhang and Goodchild, 2002). This can be shown mathematically by the following expression:

\[
 u(x_i) = U_k \text{ if } \mu_k(x_i) = \mu_{\max}(x_i) = \max(\mu_1(x_i), \ldots, \mu_{n}(x_i)) \quad (7)
\]

where \( U_k \) is the classified layer for a particular location, \( x_i \), \( k \) is the range of the land cover classes which varies between 1 and \( n \), and \( s \) is the range of the pixels (locations) of a classified data varying from 1 to \( n \). In this approach, the maximum membership value determined by Equation 7 for a location \( x_i \) is always equal or higher than the predetermined threshold \( \alpha \).

**Study Area and Data**

The study area is located in the Perth central city, Western Australia covering an area of 13 square kilometres (Figure 2). A Landsat-7 ETM+ scene, acquired on 16 December 2001 was used in the analysis to map four major land cover classes (e.g., forest, grassland, urban, and dense urban). The image was geo referenced using a first-order polynomial interpolation with nearest-neighbour resampling. The study area was clipped from the Landsat-7 ETM+ scene using the administrative boundary of the Perth city council. The visible, near- and mid- infrared bands were used in the analysis. High resolution, digital, multi-spectral video data with a spatial resolution of two meters and an orthophoto with a spatial resolution of 2.5 m and secondary source’s spatial data such as land use map (WAPC and COP; 2000) were used to validate the training sites.

The area was selected to represent both homogeneous and heterogeneous land cover classes of typical urban areas. In the Perth city area, there are distinct land cover classes such as a forested area in Kings Park, dense urban (high built-up) in west, central and east Perth. Other land cover classes include urban (built-up) and grassland (e.g., recreational spaces). A description of the land cover classes is presented in Table 1.

**Methodology**

As illustrated in Figure 1, the fuzzy supervised classification approach applied in this research comprised four main steps: preliminary data processing, fuzzy classification, integration of fuzzy memberships and defuzzification, and accuracy assessment. These steps are summarised below.

**Preliminary Data Processing**

This stage of the research comprised the selection of training areas representative of each of the land cover types, signature evaluation, and selection of the best band combination to be used in the fuzzy classification. During the training phase, representative samples of the predetermined classes (Table 1) were selected from the satellite data, based on the reference data (e.g., orthophoto and land use database).

Descriptive class statistics, including mean, standard deviation, and covariance matrix were generated from the training samples. Then, transformed divergence analysis was applied to statistically evaluate the class signature separability and select the best band combination on which to apply the fuzzy c-means algorithm for generating fuzzy memberships of the predetermined land cover classes. The transformed divergence analysis can be computed on any combination of bands, enabling to exclude the bands, or combinations thereof, that as a function of spectral class separation may not yield or correct classification result.

The transformed divergence \( (TD) \) is a modification of the divergence measure \( (D) \). It provides a prior probability of correct classification using the statistical separability based on the degree of overlap of the probability distributions between a pair of spectral classes (Richards and Jia, 1999). Thus, the larger the transformed divergence, the greater the statistical distance between the training signatures, and the higher the probability of correct classification.

**Table 1. Definitions of the Land Cover Classes Used in the Analysis**

<table>
<thead>
<tr>
<th>Land Cover Classes</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forest</td>
<td>Characterised by a wide variety of native plants, either individually or in various combinations of Eucalyptus gomphocephala, E. calophyllum, E. marginata open forest to woodland.</td>
</tr>
<tr>
<td>Grassland</td>
<td>Grassland is dominated by high percentage of grasses, shrubs, herbs and occasional native plants, which characterise recreational areas of the city.</td>
</tr>
<tr>
<td>Urban</td>
<td>Approximately 50 to 80 percent construction materials with large open roofs as well as large open transportation facilities, e.g., parking lots, and multi-lane freeways with certain amount of vegetation cover (20 percent), internal roads, rail line etc.</td>
</tr>
<tr>
<td>Dense Urban</td>
<td>Approximately 80 to 100 percent construction materials typically commercial buildings.</td>
</tr>
</tbody>
</table>
It can be computed from the following formula (Swain and Davis, 1978):
\[
D_{ij} = \frac{1}{2} tr[(V_i - V_j)(V_i^{-1} - V_j^{-1})] + \frac{1}{2} tr[(V_i^{-1} - V_j^{-1})(M_i - M_j)(M_i - M_j)^T] \\
TD_{ij} = 2000 \left(1 - \exp\left(-\frac{D_{ij}}{8}\right)\right) \\
\]
where \(i\) and \(j\) = the two signatures (classes) being compared, \(V_i\) = the covariance matrix of signature \(i\), \(M_i\) = the mean vector of signature \(i\), \(tr\) = the trace function (matrix algebra), and \(T\) = the transposition function.

Equation 9 indicates that transformed divergence increases as the distance between the classes increases, usually varying between 0 and 2000. According to Jensen (1996), a transformed divergence value of 2000 suggests an excellent class separation, 1900 provides a good separation, while values below 1700 represent a poor separability between classes. This technique has been applied in several studies to evaluate the separability of informational classes and select the best band combination to be used in the subsequent image classification (Metternicht, 1996; Metternicht and Zink, 1997 and 1998).

Measures of statistical separability containing the average and minimum divergence for every class pair and band combinations were generated using the class signatures, and the best average separabilities recorded for all band combinations are presented in Table 2. Table 2 shows that Landsat-7 ETM+ bands 1, 3, 4 and 7 provided the highest average separability (e.g., 1983) among the predetermined land cover classes. Accordingly, these four bands were selected to carry out the fuzzy classification using the fuzzy c-means algorithm previously described.

**Fuzzy Classification**

The supervised approach of the fuzzy c-means algorithm (Key et al., 1989) was implemented using Arc Macro Language© (AML) in Arc/Info® (ESRI, 2000) to calculate the fuzzy memberships for each pixel. In this algorithm, spectral values containing several pixels of the area were considered as data vectors and, using the known class means, a fuzzy membership matrix (\(c \times n\), where \(c\) and \(n\) represent number of land cover classes and data vectors, respectively) was generated. In this approach, the memberships are still a function of the weighted distance to the class means, but the algorithm no longer determines the class means. Thus, a modified algorithm was used and the memberships were generated using the following equation (Key et al., 1989):
\[
\mu_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{d_{ik}}{d_{jk}}\right)^{\gamma}} \\
\]
where \(d_{ik}\) is the distance between each observation \(x_i\) and class means \(v_k\), \(m\) is the weighting component. The measure of dissimilarity \(d_{ik}\) was calculated by the following equation:
\[
d_{ik}^2 = |x_i - v_k|^2 = (x_i - v_k)^2 A(x_i - v_k) \\
\]
where \(A\) is the inner product norm which controls the shape of the clusters generated by the fuzzy c-means algorithm. Bezdek (1981) provides details on the shape of the clusters.

The program was run to generate a fuzzy membership for each of the selected bands (e.g., 1, 3, 4, and 7) using the Euclidean norm, a weighting component (\(m\)) equal to 2 and mean values of the four major land cover classes derived from the training samples. A total of 16 outputs were produced (e.g., a fuzzy layer for each of the four land cover types in each of the four Landsat-7 ETM+ bands selected from the transformed divergence analysis). An example of such outputs is presented in Figure 3.

**Integration of Fuzzy Memberships**

Fuzzy classified land cover data on the selected bands were integrated using the fuzzy minimum, maximum, fuzzy algebraic product, fuzzy algebraic sum, and fuzzy gamma operators. Several \(\gamma\) values (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 0.95) were tested on the fuzzy gamma operator for all land cover classes considered. The various map-algebra functions (e.g., \(MAX\), \(MIN\), \(ADD\), \(SUBSTRACT\), \(MULTIPLY\), \(DIVIDE\), \(POW\)) available in Grid module of Arc/Info® (ESRI, 2000) were used to implement Equations 2, 3, 4, 5, and 6 for integrating the fuzzy memberships from Landsat-7 ETM+ bands 1, 3, 4 and 7. Thus, for each fuzzy operator, four fuzzy membership maps of a land cover type as obtained from the Landsat-7 ETM+ bands need to be integrated.

**Defuzzification**

The fuzzy land cover maps produced with each of the fuzzy operators were subsequently defuzzified to generate a final urban land cover map to be assessed for its accuracy. In defuzzifying the memberships, the fuzzy memberships of all land cover classes were considered for each location (pixel) and based on the maximum membership value, each pixel was assigned to their corresponding land cover class (crisp). That relationship is
\[
X_{\text{class}} = \text{MAX}(\mu_{A}, \mu_{B}, \mu_{C}, \ldots, \mu_{n}) \\
\]
where pixel \(X\) adopts the class label corresponding to the land cover class (e.g., A, B, C, . . . , n) where it records the highest fuzzy membership value. The map-algebra functions (e.g., \(MAX\), \(CON\), \(ADD\)) of Grid module of Arc/Info® (ESRI, 2000) were applied for defuzzification of the land cover maps. An example of defuzzified land cover map along with band 3 of Ikonos data (acquired on 29 July 2000) in grey scale is presented in Figure 4a and 4b for visual comparison between the resulting classification and the land covers in the Ikonos image.

**Accuracy Assessment**

To assess the accuracy of the defuzzified land cover maps, obtained for each of fuzzy operators described in previous sections, which are comprised of four urban land cover classes, some 200 samples sites were randomly selected from the study area and their corresponding easting and northing were identified. Using the easting and northing plane coordinates, the sample sites were plotted in a map and their position found in the field using a hand held GPS. For each site a digital photograph was taken and a class was assigned based on the dominance of a particular land cover class.
type, and these data were used in the accuracy assessment analysis in the conventional error matrix.

Commonly-used accuracy measures are shown in a typical error matrix of n number of pixels classified into c classes (Table 3). The producer’s accuracy indicates the probability of a reference pixel being correctly classified and it is related to the omission error (see Table 3). Similarly, the user’s accuracy indicates the probability of a pixel classified on the image actually represents that category on the ground (Story and Congalton, 1986) and measures the commission error. The overall accuracy of a classified image is calculated by the ratio of the sum of the pixels that form the diagonal of the matrix to the total number of pixels of samples.

In addition to the traditional accuracy measures mentioned above, Kappa analysis (Cohen, 1960; Congalton and Mead; 1983, Stehman, 1996) which is a discrete multivariate technique (Bishop et al., 1975) was used to assess the accuracy of the classification outputs obtained by applying different fuzzy operators. The results of performing a Kappa analysis is the $K_{hat}$ statistic which was measured based on the difference between the measured agreement in the error matrix and the agreement attributed to chance (Congalton and Mead, 1983; Rosenfield and Fitzpatrick-Lin, 1986; Rosenfield, 1981). This can be expressed by the following equation:

$$K_{hat} = \frac{P_o - P_c}{1 - P_c}$$

(13)

where $P_o$ is the actual agreement and $P_c$ is the chance of agreement (Congalton and Green, 1999). The above equation can be simplified in the context of error matrix described in Table 3 and expressed as follows:

$$K_{hat} = \frac{n \sum_{i=1}^{c} n_{ii} - \sum_{i=1}^{c} n_{i\cdot} n_{\cdot i}}{n^2 - \sum_{i=1}^{c} n_{i\cdot} n_{\cdot i}}$$

(14)

where $n_{ii}$ is the diagonal of the error matrix, $n$ is the total sample, $n_{i\cdot}$ is the row total and $n_{\cdot i}$ is the column total. In addition to the $K_{hat}$ statistic, conditional Kappa was also used in this analysis to determine the agreement for an individual category within the error matrix (Congalton and Green, 1999).

Accordingly, the defuzzified land cover maps produced by the fuzzy operators were compared with the field data and the accuracy measures were computed. The accuracy measures, such as, overall accuracy, producer’s and user’s accuracy, conditional Kappa coefficient, and $K_{hat}$ statistic for each defuzzified land cover map were extracted from the error matrix and a summary of accuracy measures is presented in Table 4. It should be noted that the accuracy measures of the defuzzified land cover maps resulting from the use of a fuzzy gamma operator with $\gamma$ values of 0.2, 0.3, 0.4, 0.5, and 0.6 were not included in Table 4 as they were very similar with the accuracy measures of $\gamma$ value of 0.1 and 0.7.

### Results and Discussion

A comparison of the accuracy measures (Table 4) reveals that defuzzified land cover maps generated by fuzzy algebraic
were up to 16 percent more accurate than fuzzy OR, fuzzy product and fuzzy gamma operator ranging from 0.1 to 0.8 exhibited the highest overall and accuracy. Overall accuracy and \( K_{\text{stat}} \) values for the fuzzy algebraic product and fuzzy gamma operator ranging from 0.1 to 0.8 were up to 16 percent more accurate than fuzzy OR, fuzzy AND, and fuzzy algebraic sum, while 1 percent more accurate than using a fuzzy gamma operator with a \( \gamma \) value of 0.9 and 2 percent more accurate than a \( \gamma \) value of 0.95. A further examination of the accuracy measures indicates that the omission error (related to the producer’s accuracy) of the maps obtained by using fuzzy OR and fuzzy AND, particularly for the class urban, is significantly higher than the error reported when applying fuzzy algebraic sum. Fuzzy algebraic product, or fuzzy gamma with \( \gamma \) values ranging from 0.1 to 0.95 (Table 4). This can be justified by the fact that fuzzy OR considers the highest membership among the layers being integrated, while fuzzy AND selects the minimum fuzzy membership value recorded for a particular land cover class. Accordingly, fuzzy memberships of land cover classes generated by the fuzzy OR and fuzzy AND utilised a single fuzzy value of the selected Landsat-7 ETM+ bands (e.g., the output membership is controlled by the maximum or minimum value of the input bands). Thus, there was no compensation of information among the selected bands which led to misclassification. Likewise, commission errors occur on the class grassland and dense urban derived from the fuzzy algebraic sum. Although fuzzy algebraic sum considers all the fuzzy membership values of the bands used in the analysis, it still tended to assign the maximum membership to each pixel (location), creating an overestimation of the membership of grassland and dense urban which contributed to the misclassification.

The results of fuzzy algebraic product and fuzzy gamma operator with \( \gamma \) values ranging from 0.1 to 0.95 are very encouraging as the accuracy measures were found higher than those obtained by using fuzzy OR, fuzzy AND and fuzzy algebraic sum. Seemingly, the high accuracy is attributed to the ability of fuzzy algebraic product and the gamma operators of considering the fuzzy memberships obtained from all the selected Landsat-7 ETM+ bands for each of the land cover types classified. The accuracy measures obtained from using a fuzzy gamma operator with \( \gamma \) values ranging from 0.1 to 0.8 suggest the dominant contribution of the fuzzy algebraic product (see the Fuzzy Gamma Operation section) which produced similar accuracies for the land cover classes considered. However, the producer’s accuracy for the class urban decreased when using a fuzzy gamma operator with a \( \gamma \) value higher than 0.8 indicating that as the \( \gamma \) value increases beyond 0.8, the land cover map tended to be the same as the fuzzy algebraic sum (see Equation 6). Thus, the fuzzy algebraic product and fuzzy gamma operator with \( \gamma \) values ranging from 0.1 to 0.8 seem to offer the most appropriate balance for integrating the fuzzy memberships of urban land covers computed on selected Landsat-7 ETM+ bands, which provided the highest overall accuracy (82 percent) and \( K_{\text{stat}} \) statistic (75 percent).

An examination of the producer’s accuracy and user’s accuracy of the defuzzified land cover maps generated using the fuzzy algebraic product and fuzzy gamma operator with \( \gamma \) values ranging from 0.1 to 0.8 reveals accuracies above 80 percent for forest and dense urban classes. Among the land covers, a relatively high commission error (19 percent) occurred in dense urban which was mostly confused with urban areas. A significant disagreement was found between the defuzzified land cover maps and the field data for grassland and urban areas. Accuracy measures, particularly the user’s accuracy, tended to be between 73 percent and 76 percent for grassland and urban areas respectively. Notably, high commission errors occurred for grassland (27 percent) which was mostly confused with forest followed by urban area. The commission errors of urban areas (24 percent) are attributed to their classification as dense urban or grassland.

---

Table 3. A Typical Error Matrix Showing Commonly Used Accuracy Measures

<table>
<thead>
<tr>
<th>Classified Data</th>
<th>Reference Data</th>
<th>Row Total ( (n_{ij}) )</th>
<th>User’s Accuracy (%)</th>
<th>Commission (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>c</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( n_{11} )</td>
<td>( n_{12} )</td>
<td>( n_{1c} )</td>
<td>( n_{1} )</td>
</tr>
<tr>
<td>2</td>
<td>( n_{21} )</td>
<td>( n_{22} )</td>
<td>( n_{2c} )</td>
<td>( n_{2} )</td>
</tr>
<tr>
<td>C</td>
<td>( n_{c1} )</td>
<td>( n_{c2} )</td>
<td>( n_{cc} )</td>
<td>( n_{c} )</td>
</tr>
<tr>
<td>Column Total ( (n_{i}) )</td>
<td>( n_{1} )</td>
<td>( n_{2} )</td>
<td>( n_{c} )</td>
<td>( n )</td>
</tr>
<tr>
<td>Producer’s Accuracy (%)</td>
<td>( n_{1}/n_{1i} )</td>
<td>( n_{2}/n_{2i} )</td>
<td>( n_{c}/n_{ci} )</td>
<td></td>
</tr>
<tr>
<td>Omission (%)</td>
<td>( 1-n_{1}/n_{1i} )</td>
<td>( 1-n_{2}/n_{2i} )</td>
<td>( 1-n_{c}/n_{ci} )</td>
<td></td>
</tr>
</tbody>
</table>

PHOTOGRAAMMETRIC ENGINEERING & REMOTE SENSING
TABLE 4. SUMMARY OF ACCURACY MEASURES OF THE DEFUZZIFIED LAND COVER MAPS

<table>
<thead>
<tr>
<th>Accuracy Measures</th>
<th>Forest</th>
<th>Grassland</th>
<th>Urban</th>
<th>Dense Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy OR</td>
<td>Producer’s Accuracy (%)</td>
<td>87</td>
<td>72</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>72</td>
<td>68</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>56</td>
<td>61</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>70</td>
<td>59</td>
<td>61</td>
</tr>
<tr>
<td>Fuzzy AND</td>
<td>Producer’s Accuracy (%)</td>
<td>88</td>
<td>62</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>82</td>
<td>82</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>71</td>
<td>78</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>73</td>
<td>64</td>
<td>61</td>
</tr>
<tr>
<td>Fuzzy Algebraic Product</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>87</td>
<td>69</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>70</td>
<td>75</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>79</td>
<td>74</td>
<td>68</td>
</tr>
<tr>
<td>Fuzzy Algebraic Sum</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>87</td>
<td>69</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>80</td>
<td>65</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>82</td>
<td>75</td>
<td>71</td>
</tr>
<tr>
<td>Fuzzy Gamma Operation ($\gamma = 0.1$)</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>89</td>
<td>73</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>84</td>
<td>68</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>82</td>
<td>75</td>
<td>71</td>
</tr>
<tr>
<td>Fuzzy Gamma Operation ($\gamma = 0.7$)</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>89</td>
<td>73</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>84</td>
<td>68</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>82</td>
<td>75</td>
<td>71</td>
</tr>
<tr>
<td>Fuzzy Gamma Operation ($\gamma = 0.8$)</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>89</td>
<td>73</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>84</td>
<td>68</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>82</td>
<td>75</td>
<td>71</td>
</tr>
<tr>
<td>Fuzzy Gamma Operation ($\gamma = 0.9$)</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>89</td>
<td>73</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>84</td>
<td>68</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>82</td>
<td>75</td>
<td>74</td>
</tr>
<tr>
<td>Fuzzy Gamma Operation ($\gamma = 0.95$)</td>
<td>Producer’s Accuracy (%)</td>
<td>90</td>
<td>76</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>User’s Accuracy (%)</td>
<td>87</td>
<td>73</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Conditional Kappa Coefficient (%)</td>
<td>80</td>
<td>68</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Overall Accuracy (%)</td>
<td>80</td>
<td>73</td>
<td>74</td>
</tr>
</tbody>
</table>

Similar variations of omission errors were found for grassland and urban area. The maximum omission error is repeated for the urban area (30 percent), mostly confused with dense urban followed by grassland and forest. Likewise, grassland was wrongly labelled as urban followed by forest.

Several sources of disagreement between classified and reference data are identified. First, a possible source of disagreement may be due to the similar spectral response of native vegetation comprising the forest and grassland areas. As mentioned from Table 2, the class grassland is characterised by a mixture of grass, shrubs, and occasional native plants. Second, the seasonal effect on the spectral behaviour of the class grassland, as the satellite imagery was acquired during summer, when grassland and some shrubs tend to dry up due to the lack of rainfall in the area (e.g., most of rain falls in winter), and their spectral response tends to be similar to soils (e.g., reduction of the characteristic absorption feature of vegetation in band 3 and high reflectance in the near infrared band 4) and other man-made features which might lead to a labelling as urban. Another significant source of disagreement may result from the spectral similarity between urban and dense urban which lead to misclassification.

The overall accuracy (82 percent) and $K_{hat}$ statistic (75 percent) of the defuzzified land cover maps generated by fuzzy algebraic product and the gamma ($\gamma$) operator with $\gamma$ values ranging from 0.1 to 0.8 tended to show strong agreement with the reference data. This provides an indication that a fuzzy supervised classification as implemented in this research has a strong potential to classify complex urban land cover classes using multi-spectral satellite data.

Summary and Conclusions
This paper describes the procedure adopted for implementing a fuzzy supervised classification of multispectral remote sensing data acquired over a heterogeneous urban land cover characterising the metropolitan area of Perth, Western Australia. While previous research has focused on the use of the fuzzy c-means algorithm on single data layers, we present a methodology for applying the algorithm on multispectral data sets. To this end, a transformed divergence analysis is applied for selecting the best combination of multispectral
bands to be fuzzified, and a variety of fuzzy operators (e.g., fuzzy minimum, maximum, algebraic product, algebraic sum, and fuzzy gamma) are tested for integrating fuzzy membership values of a class associated with multiple spectral bands.

The major findings of this research can be summarised as follows:

1. The transformed divergence analysis, used in traditional crisp classification for establishing a priori the upper bound achievable on classification accuracy for an existing set of spectral classes as explained in Richards and Jia (1999) provided a sound basis for selecting the most appropriate spectral bands of the Landsat-7 ETM+ that were subsequently fuzzified for the production of a urban land cover map. The highest average separability among the four land cover types considered in this research (e.g., forest, grassland, urban, and dense urban) was provided by a combination of Landsat-7 ETM+ bands 1, 3, 4 and 7;

2. The approach of assigning a land cover class (crisp value) in each location (e.g., pixel) using the maximum fuzzy membership value of the class(es) present within a pixel, facilitated the process of defuzzification for accuracy assessment analysis. This approach of defuzzification is based on the approaches discussed by Zadeh (1968) and Zhang and Goodchild (2002);

3. The accuracy of the land cover maps generated by integrating fuzzy membership values associated with multiple spectral bands using various fuzzy operators reveals that the fuzzy minimum operator (e.g., fuzzy AND) and fuzzy maximum operator (e.g., fuzzy OR) are not suitable for integrating the fuzzy class memberships computed from selected Landsat-7 ETM+ bands, as these operators do not offer any compensation of information amongst the bands used in the analysis (e.g., a single value, whether maximum or minimum) determines the fuzzy membership of a pixel to the classes considered. Accordingly, the lowest conditional Kappa coefficient, overall accuracy and Kstat statistic correspond to the implementation of these fuzzy operators. When applying fuzzy operators such as fuzzy algebraic sum, fuzzy algebraic product and fuzzy gamma (with varying γ values) all contributing membership values computed in the selected Landsat-7 ETM+ spectral bands have an effect on the result, increasing the accuracy in the identification of the land cover types. Furthermore, analysis of the accuracy assessment results show that the fuzzy algebraic product and the fuzzy gamma operators, the later with γ values varying between 0.1 to 0.8, outperformed the results obtained by applying fuzzy algebraic sum. These two fuzzy operators produced overall classification accuracies above 80 percent, being therefore recommended, as the optimal fuzzy operators for integrating fuzzy membership values associated with multiple spectral bands.

Acknowledgments

Spatial thanks is given to Dr. Richard Smith and Mr. Peter Sanders of the Satellite Remote Sensing Services (SRSS), Leuwin Centre for Earth Sensing Technologies, Perth, Western Australia for the provision of the satellite data set used in this research. The authors also wish to acknowledge the comments of the three anonymous reviewers which helped to improve the quality of the paper.

References


Western Australian Planning Commission (WAPC) and City of Perth (COP), 2000. *Future Perth: Central city indicators*, State of Western Australia and City of Perth.


